

S-5/PHSH/DSE-II/19

TDP (Honours) 5th Semester Exam., 2019

PHYSICS

(Honours)

PAPER - DSE - II

(Advanced Mathematical Physics)

Full Marks : 80

Time : 3 Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

1. Answer any *eight* questions :

$2 \times 8 = 16$

(i) What is the value of ' a ' if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix ?

(ii) Find the eigen value of the matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

[Turn Over]

(2)

- (iii) State the rule by which a covariant tensor can be transformed. Write properties of Kronecker delta.
- (iv) Find the probability of getting 4 heads in six tosses of a fair coin.
- (v) Write the range of probability distribution for all possible values of a random variable. If the standard deviation of Poisson distribution is $\sqrt{2}$, then find probability for $r = 2$.
- (vi) If α be a non-zero vector element in V and a, b are scalars in F , then what will be relation between a and b ?
- $\{\alpha_i\}$ set of vectors are linearly dependent or not?
- (vii) If A^i and B_j are the contravariant and covariant tensor of rank 1, then what will be $A^i B_j$?
- (viii) Write the inverse of $-i$ in the multiplicative group $\{1, -1, i, -i\}$.
- (ix) Find $\epsilon_{ilm} \epsilon_{jlm}$. What will be the identity element in the group $G\{2, 4, 6, 8\}$ under multiplication modulo 10?
- (x) The set of integers ' z ' with the binary operation $**$ defined as $a*b = a + b + 1$ for $a, b \in z$ is a group. Write the identity element of this group.

(3)

Section - B

Answer the following questions : $16 \times 4 = 64$

2. (a) (i) Show that every square matrix can be uniquely expressed as the sum of a symmetric and skew symmetric matrix.
- (ii) State and explain Cayley-Hamilton theorem.
- (iii) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

- (iv) Distinguish between symmetric and skew symmetric matrices with example.

$$4+4+4+4$$

Or

- (b) (i) Show that the matrix

$$A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$

is orthogonal in character.

[Turn Over]

(4)

- (ii) Solve the following equations with the help of matrices —

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

- (iii) Write the condition of a matrix to be a Hermitian. Show that the eigen values of a Hermitian matrix are real.

- (iv) Find the values of x, y, z and a which satisfy the matrix equation :

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix} \quad 3+4+5+4$$

3. (a) (i) Write the condition for two vectors to be linearly dependent. Show that the vectors of the set $(1, 2, 3, 4), (1, 0, 2, 1), (2, -1, 4, 1), (-1, 3, 0, 4)$ are linearly dependent.

- (ii) What is contravariant and co-variant tensors? Show that the law of transformation for a contravariant vector is transitive.

- (iii) Define vector space and its basic with example.
(2+5)+(2+3)+4

(5)

Or

- (b) (i) State and explain quotient law of a tensor.

- (ii) If $A_{\lambda\mu}$ is a skew symmetric tensor, show that $(B_{\gamma}^{\mu} + B_{\tau}^{\sigma} + B_{\tau}^{\mu} + B_{\gamma}^{\sigma})A_{\mu\sigma} = 0$.

- (iii) If a mixed tensor A_j^i is symmetric in the x' co-ordinate system, find out whether it will be symmetric in any other co-ordinate system or not.

- (iv) Find the coordinates of the vector $\alpha = (2, 3, 1)$ relative to the basis

$$\beta = \{(1, 1, 1), (1, 1, 0), (0, 1, 0)\}. \quad 4+4+4+4$$

4. (a) (i) Under what condition a group will be a cyclic group? Write down the characteristics of a cyclic group.

- (ii) Show that the C_{4v} is homomorphic to the group $(1, -1)$ under multiplication. Also show that this 4 to -1 homomorphic mapping can be established in 3 distinct ways.

[Turn Over]

(6)

- (iii) What do you mean by finite and infinite group? Define order of such group.

$$(2+5)+(2+3)+4$$

Or

- (b) (i) What do you mean by homomorphism and isomorphism of groups? Write down their properties.

- (ii) Show that every subgroup of index 2 is a normal subgroup.

- (iii) Obtain the direct products of all the irreducible representation of the group C_{3v} and reduce them into direct sums of the irreducible representation.

- (iv) What do you mean by Kernel of the group? Explain with example.

$$(2+4)+3+4+3$$

5. (a) (i) State and prove Baye's theorem of probability.

- (ii) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(7)

- (iii) An urn contains 10 black balls and 10 red balls. What is the probability of drawing the two balls of the same colour.

- (iv) Explain conditional probability with example.
 $(2+3)+4+4+3$

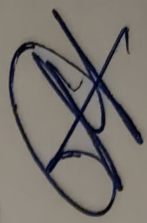
Or

- (b) (i) What is co-variance? Show that the co-variance of two independent variables is always zero.

- (ii) What is Poisson's distribution? Find the expression for it.

- (iii) What is the probability that a leap year selected at random contains 53 Sundays?

- (iv) What do you mean by "probability distribution of a random variable"? Name three important probability distribution.
 $(2+2)+(2+3)+4+3$



S-5/PHSH/DSE-II/21

TDP (Honours) 5th Semester Exam., 2021

PHYSICS

(Honours)

PAPER : (DSE-II)

Full Marks : 80

Time : 3 Hours

The figures in the margin indicate full marks.

SECTION—A

1. Answer any **eight** questions : 2×8=16

(a) Define linear vector space and give an example of this.

(b) Identify if the matrix $u = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$ is unitary or not.

(c) $A(p, q, r)B_r^{qs} = C_p^s$, here B_r^{qs} and C_p^s are tensors. What will be the nature of $A(p, q, r)$ i.e., is scalar, vector or tensor?

(d) Define a group.

(e) What is inner product (and when an inner product space is called unitary space?)

(2)

Define a priori probability.

(g) What do you mean by linear dependence and independence of vectors?

(h) If a pair of dice is thrown, what is the probability that the sum of 8 shows?

(i) Define expectation and variance of random variable.

(j) What do you mean by basis of a vector space?

SECTION—B

(There are **four** questions from Question No. 2 to Question No. 5. Answer either (a) or (b) from each question.)

2. (a) (i) Solve the following equations in matrix method :

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8$$

(3)

(ii) Express

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$$

as the sum of a symmetric and skew-symmetric matrix.

(iii) Show that eigenvalues of a Hermitian matrix are real.

(iv) Find the inverse of the following matrix :

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{pmatrix}$$

$$4+4+4+4=16$$

(OR)

(b) (i) Determine the characteristic roots and characteristic vectors of the following matrix :

$$B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

(ii) What is diagonal matrix? Diagonalize the following matrix :

$$A = \begin{bmatrix} -1 & -2 & 2 \\ 4 & 3 & -4 \\ 0 & -2 & 1 \end{bmatrix}$$

(4)

(iii) Show that $\text{Trace}(AB) = \text{Trace}(BA)$

(iv) State Cayley-Hamilton theorem. Hence find the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

$$4+5+3+4=16$$

3. (a) (i) Show that the three-dimensional vectors

$$x^1 = (3, 0, 3)$$

$$x^2 = (0, 3, 0)$$

$$x^3 = (3, 0, 3)$$

are linearly independent.

(ii) Identify if the vector $(2, -5, 3)$ can be expressed as a linear combination of vectors

$$a_1 = (1, -3, 2)$$

$$a_2 = (2, -4, -1)$$

$$a_3 = (1, -5, 7)$$

If possible, identify if they are forming a linearly dependent or independent relation.

(iii) Define a tensor in 4-D space and from this, explain the rank of a tensor with example.

(5)

(iv) "The sum and difference of two tensors of the same rank results a third tensor of the same rank". Prove this.

$$3+(2+3)+(2+2)+4=16$$

(OR)

(b) (i) What do you mean by change of basis? Consider $\hat{A}$ and $\hat{B}$ are two linear operators in the vector space V . Write the basic rule to be followed by the linear operators $\hat{A}$ and $\hat{B}$.

(ii) Prove that if A^μ and B_μ are covariant and contravariant vectors, then $A^\mu B_\mu$ is invariant. What are Kronecker alternating tensors?

(iii) Show that velocity is a contravariant tensor of rank one.

(iv) Prove that δ_q^p is a mixed tensor of second rank.

(v) If a tensor B^{pq} is symmetric in one coordinate system, show that it remains symmetric with respect to p and q in any coordinate system.

$$4+(3+1)+2+3+3=16$$

[Turn Over]

(6)

4. (a) (i) Explain character of representation with example.
- (ii) What do you mean by centre of a group?
- (iii) Write the basic properties of a group. Name two trivial subgroups of a group.
- (iv) Define cyclic group and prove that every subgroup of a cyclic group is cyclic. What is permutation group?
- (v) What is a set?

$$3+2+(2+2)+(3+2)+2=16$$

(OR)

- (b) (i) Show that in a group $(G, \circ)$ every element possesses a unique inverse.
- (ii) What is irreducible representation? When a representation T is said to be fully reduced?
- (iii) When is mapping called external binary operation? Write some properties of binary operation.
- (iv) If $(G, \circ)$ be a group and $a^2 = e \forall a \in G$, then show that the group must be commutative.

$$4+(2+2)+(2+3)+3=16$$

[Continued]

(7)

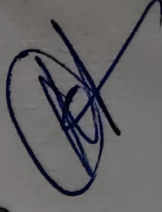
5. (a) (i) Explain the terms conditional probability and variance.
- (ii) Find the expected number of heads when n coins are tossed.
- (iii) Prove that for any two events A and B , the probability that either A or B or both occur can be given by
- $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- (iv) In a sum of $2n$ observations, half of them are equal to a and remaining half are equal to $-a$. If $SD = 2$, find $|a|$.

$$4+4+4+4=16$$

(OR)

- (b) (i) Define mean, median and mode.
- (ii) For any two events, such that $A \subseteq B$, then prove $P(A) \leq P(B)$.
- (iii) Mean of 100 items is 49. It was found that three terms which should have been 60, 70, 80 were wrongly read as 40, 20 and 50 respectively. Find the correct number of mean.
- (iv) What is normal distribution or Gaussian distribution? Write the probability function and draw the distribution curve.

$$4+4+4+4=16$$



S-5/PHSH/DSE-II/22

**TDPH (Honours) 5th Semester
Supplementary Exam., 2022**

PHYSICS

(Honours)

PAPER : DSE – II

Full Marks : 80

Time : 3 Hours

Answer **all** questions from **both** the Sections.

The figures in the margin indicate full marks.

Candidates are required to give their answers
in their own words as far as practicable.

SECTION—A

1. Answer any **eight** questions : 2×8=16

(a) What is the inverse of $-i$ in the multiplicative group $\{1, -1, i, -i\}$?

(b) A set of all non-singular matrices forms a group under which operation?

(c) Define trace of a matrix.

~~(d)~~ What is tensor?

~~(e)~~ A pair of coin has thrown. Find the probability of getting two tails.

(f) When is a matrix Hermitian?

PHSH-5/660

[Turn Over]

(2)

- (g) What do you mean by group multiplication table?
- (h) Define isomorphism.
- (i) How can someone calculate the expectation value?
- (ii) What is basis of a vector space?

SECTION-B

There are four questions from Question No. 2 to Question No. 5. Answer either (a) or (b) from each question given below :

2. (a) (i) Define an orthogonal matrix. Show that the product of two orthogonal matrices is orthogonal.

- (ii) Find the value of A when

$$\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} A = \begin{pmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{pmatrix}$$

- (iii) What is the inverse of the matrix A given below?

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{pmatrix}$$

- (iv) Write an eigenvalue equation of a matrix and identify the eigenvalue.

$$(2+4)+4+4+2=16$$

[Continued]

(3)

(OR)

- (b) (i) State and explain Cayley-Hamilton theorem.
- (ii) Show that the eigenvalue of an anti-Hermitian matrix is imaginary.
- (iii) Find the inverse of the following matrix :

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{pmatrix}$$

- (iv) Determine the characteristic roots and characteristic vectors of the following matrix :

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{pmatrix}$$

$$4+4+4+4=16$$

3. (a) (i) What do you mean by linear dependence of two vectors? Explain with example.

- (ii) Show that the vectors $(1, -2, 3)$, $(2, 3, 1)$ and $(-1, 3, 2)$ form a basis of the vector space V_3 over real numbers.

- (iii) Identify if the following set of vectors is linearly dependent or not :
 $(1, 2, 3, 4)$, $(1, 0, 2, 1)$, $(2, -1, 4, 1)$ and $(-1, 3, 0, 4)$.

PHSH-5/660

[Turn Over]

PHSH-5/660

(4)

- (iv) What is vector space? Explain the algebra followed by a vector space.
 $3+4+5+4=16$

(OR)

- (b) (i) Define covariant and contravariant tensors with example.

- (ii) What do you mean by the inner product? Explain with example.

- (iii) Show that the vectors $(1, 1, 2)$, $(3, 5, 2)$ and $(1, 0, 0)$ are linearly independent.

- (iv) If $A^{\mu\nu}$ and $B_{\mu\nu}$ are covariant and contravariant vectors, then show that $A^{\mu\nu}B_{\mu\nu}$ is invariant.
 $4+4+4+4=16$

4. (a) (i) Prove that a group is Abelian if and only if the elements of the group are isomorphic to their inverse.

- (ii) What are group and subgroup? What are the trivial subgroup of a group?

- (iii) Show that in a group $(G, 0)$ every element has a unique inverse.

PHSH-5/660

[Continued]

(5)

- (iv) What is external binary operation? Write some properties of binary operation.
 $4+4+4+4=16$

(OR)

- (b) (i) What is a cyclic group? Prove that every subgroup of a cyclic group is cyclic.

- (ii) Prove that every subgroup of an Abelian group is normal.

- (iii) Explain the characters of representation with example.

- (iv) Prove that the identity of a subgroup is the same as group.
 $(1+3)+4+4+4=16$

5. (a) (i) What is random variable? How can you explain the probability distribution of a random variable?

- (ii) Define variance and covariance.

- (iii) Draw the probability distribution curve of Gaussian and Poisson distributions. Write the meanings of the different terms of the distribution function.

- (iv) Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.
 $(2+2)+4+(2+2)+4=16$

(OR)

- (b) (i) Find the expected number of tails when n coins are tossed.

[Turn Over]

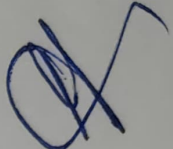
PHSH-5/660

(6)

- (ii) Prove $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Symbols have their usual meanings.
- (iii) Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$. Find $P(E | F)$ and $P(F | E)$.
- (iv) What is standard deviation? For $\bar{x}$ there are three measured values x_1 , x_2 and x_3 , find the standard deviation.

$$4+4+4+(2+2)=16$$





S-5/PHSH/DSE-II/22

TDP (Honours) 5th Semester Exam., 2022

PHYSICS

(Honours)

SECOND PAPER : DSE – II

Full Marks : 80

Time : 3 Hours

Answer from **both** the Sections as directed.

The figures in the margin indicate full marks.

Candidates are required to give their answers
in their own words as far as practicable.

SECTION—A

1. Answer any **eight** of the following questions :

2×8=16

(a) Given, $A = \begin{bmatrix} 1 & -1 & -1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

Find AB .

(b) What do you mean by vector space?

(c) Define singular and non-singular matrix.

(d) State and explain quotient law.

PHSH-5/882

[Turn Over]

(e) The set of vectors is given as $\{(a, b) \in \mathbb{R}^2 : b = 4a + 1\}$. Identify if the given set is a vector space or not.

(f) Is it possible to diagonalize the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$? Give the reason.

(g) Is the inverse element of an element in a group is unique? Justify logically.

(h) A subset S of G is a subgroup of a group $(G, *)$; then show that for any pair of elements $a, b \in S$, $a * b^{-1} \in S$

(i) Two dice are tossed. Find the probability so that the total score is a prime number.

(j) What is the utility of different statistical distribution?

SECTION—B

There are four questions from Question No. 2 to Question No. 5. Answer either (a) or (b) from each question given below :

$$16 \times 4 = 64$$

2. (a) (i) Show that eigenvectors of a Hermitian matrix corresponding to two distinct eigenvalues are orthogonal.

PHSH-5/882

[Continued]

(3)

(ii) Solve the given set of equation

$$x + z = 4$$

$$2x + y + z = 7$$

$$x + y + 2z = 9$$

(iii) Find the eigenvalue and eigenvector of the given matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$$

(iv) Determine if the given matrix is singular or not. If it is non-singular, find the inverse :

$$\begin{bmatrix} 6 & 0 \\ -3 & 2 \end{bmatrix}$$

$$4+4+4+4=16$$

(OR)

(b) (i) Define a singular matrix. For what value of x , the matrix A is singular

$$\text{where } A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

What is the physical significance of singular matrix?

(ii) Verify Cayley-Hamilton theorem for the

$$\text{matrix } B = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \text{ and find } B^{-1}.$$

PHSH-5/882

[Turn Over]

(4)

(iii) What does it physically mean that the two matrices are orthogonal?
If λ is an eigenvalue of an orthogonal matrix A , then show that $1/\lambda$ is also eigenvalue of the same matrix A .

(iv) Show that the transpose of the product of two matrices is the product of their transposes in reverse order.

$$(1+3+1)+4+(1+3)+3=16$$

3. (a) (i) What do you mean by abstract binary operation?

(ii) Let $V = \mathbb{R}^+$ be a set of all positive reals. The operation addition in the set is defined as $\vec{x} + \vec{y} = \vec{x} \cdot \vec{y}$ where $x, y \in \mathbb{R}^+$. The scalar multiplication by a scalar K

is defined as $K\vec{x} = x^K$ $x \in \mathbb{R}$.

Show that V is a vector space.

(iii) If V is a vector space and u_1 and u_2 are two subspaces. Find the common point in the intersection of two subspaces. Identify if the vectors V_1 , V_2 and V_3 are linearly dependent or not where

$$V_1 = (1, -2, 3), V_2 = (5, 6, -1)$$

$$\text{and } V_3 = (3, 2, 1).$$

(5)

(iv) Show that the set $B = \{u_1, u_2\}$ is a basis vector for $\mathbb{R}^2$ where $u_1 = (3, 4)$ and $u_2 = (1, -3)$. $2+4+2+4+4=16$

(OR)

(b) (i) Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

(ii) Given $A(r, s, t)B^{st} = C^r$,

In the above relation C^r is a tensor B^{st} is also an arbitrary tensor. Then prove that $A(r, s, t)$ is also a tensor and can be represented as A_{st}^r .

(iii) Let $\vec{V} = \mathbb{R}^3$ and $B = \{\vec{i}, \vec{j}, \vec{k}\}$ where

$$\vec{i} = (2, 0, 0), \quad \vec{j} = (0, 2, 0), \quad \text{and}$$

$\vec{k} = (0, 0, 2)$. Identify if B is a linearly independent set in $\mathbb{R}^3$ or not. Can B be considered as basis for $\mathbb{R}^3$?

[Turn Over]

(6)

(iv) Prove that the contraction of tensor A_q^p is a scalar or invariant.

(v) Prove that $\epsilon_{ilm} \epsilon_{jlm} = 2\delta_{ij}$.

$$3+3+(3+1)+3+3=16$$

4. (a) (i) What is subgroup? What are the two trivial subgroups of a group?

(ii) Define homomorphism and isomorphism between two algebraic systems. Given an example of homomorphism.

(iii) Show that in a group $(G, *)$ the equations $a * x = b$ and $y * a = b$ have unique solutions for the unknown x and y , given by $x = a^{-1} * b$, $y = b * a^{-1}$, when $a, b \in G$.

(iv) Let Z be the group of integers with binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of group $\{Z, *\}$.

$$2+2+3+3+3+3=16$$

(OR)

(b) (i) Obtain the direct products of all the irreducible representation of the group C_{3u} and reduce them into direct sum of the irreducible representation.

[Continued]

(7)

(ii) Prove that the elements common to two intersecting subgroups G_1 and G_2 form a subgroup of group G .

(iii) Prove that the element of the two different classes are all distinct. A group comprises entire class.

(iv) Write the group multiplication table of any group and explain their importance of it.

$$4+4+3+2+3=16$$

5. (a) (i) Show that the probability of the two event and their combined probability are connected by the following relation :

$$P(A+B) = P(A) + P(B) - P(AB)$$

(ii) Show that if two events are independent, the probability of both occurring is equal to the product of the probability of each event occurring independently.

(iii) Given, $P(A) = 1/2$ and $P(A \cup B) = 3/5$. Find the value of $P(B)$ if the event A and B are mutually exclusive and independent. What do you mean by mutually independent and exclusive?

(iv) If A and A' are two complimentary events, then show that the probability of the event is equal to the difference of one and the other event.

$$4+4+(4+1)+3=16$$

[Turn Over]

(8)
(OR)

(b) (i) What do you mean by binomial law of probability?

(ii) X follows a binomial distribution with parameters $n = 8$ $P = 1/2$. Then find

$$P(|x - 4|) \leq Z.$$

(iii) What do you mean by repeated trial? Calculate the probability that an event happens at least m times in n trials.

(iv) Let A and B be independent event with $P(A) = .3$ and $P(B) = .4$. Find

$$P(A \cap B) \text{ and } P(A \cup B). \quad 4+4+4+4=16$$



(3)
S-5/PHSH/DSE-II/23

TDP (Honours) 5th Semester Exam., 2023

PHYSICS

(Honours)

SECOND PAPER : DSE - II

Full Marks : 80

Time : 3 Hours

Answer from **both** the Sections as directed.

The figures in the margin indicate full marks.

Candidates are required to give their answers
in their own words as far as practicable.

SECTION—A

1. Answer any **eight** of the following questions :

2×8=16

(a) Write a square matrix so that $A = A^T$, where
A is the matrix.

(b) If two vectors are linearly independent,
explain whether they will be orthogonal or
not.

PHSH-5/1504

[Turn Over]

(2)

- (c) Identify if the square matrix B is singular or not.

$$B = \begin{bmatrix} 2 & 5 & 19 \\ 1 & -2 & 4 \\ -3 & 2 & 0 \end{bmatrix}$$

- (d) What is the physical meaning of inner product of two tensors?

- (e) Show that the product of two contravariant vectors is a contravariant tensor?

- (f) Write the inverse of i in the multiplicative group $\{1, -1, -i, i\}$.

- (g) Define standard deviation with example.

- (h) Two dice are tossed. Find the probability so that the total score is a prime number.

- (i) What is permutation group?

- (j) Can the basis vector of a vector space be linearly dependent? Explain.

PHSH-5/1504

[Continued]

(3)

SECTION-B

There are four questions from Question No. 2 to Question No. 5. Answer either (a) or (b) from each question given below : $16 \times 4 = 64$

2. (a) (i) Find the inverse of the matrix A by using Cayley-Hamilton theorem.

$$A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

- (ii) Verify the relation $[AB]^T = B^T A^T$ using the following matrices

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

- (iii) When is a matrix said to be orthogonal? Show that the product of two orthogonal matrices is orthogonal.

- (iv) Solve the following system of equations :

$$x + 2y - z = 6$$

$$2x + 5y + 3z = 9$$

$$3x + y + 2z = 3$$

PHSH-5/1504

[Turn Over]

(4)

- (v) Write the steps to diagonalize a matrix.
 $3+3+(1+3)+4+2=16$

(OR)

- (b) (i) Find the eigenvalue and eigenvector of the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$$

- (ii) Show that the eigenvalue of a skew-Hermitian matrix is purely imaginary.

- (iii) Find the roots of the following equation:

$$\begin{vmatrix} 3x-1 & 4x & 6 \\ 2(1-x) & 1 & 3x \\ 11 & 5x & 4-3x \end{vmatrix} = 0$$

- (iv) Identify if the following matrix is unitary or not.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$5+3+5+3=16$$

3. (a) (i) Examine whether the sets are vector space.

$$\{(a, b) \in \mathbb{R}^2 : b = 3a^2 + 5\}$$

$$\{(a, b) \in \mathbb{R}^2 : b = 5a + 2\}$$

PHSH-5/1504

[Continued]

(5)

- (ii) What do you mean by contraction? Show that Kronecker delta is a mixed tensor of rank two.

- (iii) Show that if a tensor is skew symmetric with respect to a pair of indices in one coordinate system, it is so in every coordinate system.

- (iv) Find the coordinate of the vector α relative to the basis

$$b = \{(1, 1, 1), (1, 1, 0), (0, 1, 0)\}, \text{ where}$$

$$\alpha = (2, 3, 1) \quad 4+(2+2)+4+4=16$$

(OR)

- (b) (i) Show that the set of vectors $\alpha = (1, 2, -1, -2)$, $\beta = (2, 3, 0, -1)$, $\gamma = (1, 2, 1, 4)$ and $\epsilon = (1, 3, -1, 0)$ is the basis of a vector space V_4 over real numbers.

- (ii) For which value of t , the set $(\cos t, \sin t)$, $(-\sin t, \cos t)$ is linearly independent in $\mathbb{R}^2$?

- (iii) Show that a set of quantities whose inner product with an arbitrary vector is a tensor, itself a tensor.

- (iv) If A_j^i is a tensor and it vanishes in one coordinate system x^i , then show that it will vanish in all coordinate system.

$$4+4+4+4=16$$

PHSH-5/1504

[Turn Over]

(6)

4. (a) (i) Considering the example of two groups, explain how they are homomorphic and isomorphic in nature.
- (ii) Prove that the identity element of a subgroup is the same as the group.

(iii) If $\omega = (1)^{\frac{1}{3}}$, then show that the group formed by the set $\{1, \omega, \omega^2\}$ is a cyclic group of order 2.

(iv) Obtain the direct products of all the irreducible representation of the group C_{3v} and reduce them into the direct sums of the irreducible representations.

$$4+4+4+4=16$$

(OR)

(b) (i) What is external binary operation? Write some properties of binary operation.

(ii) Define group and subgroup with example. When is a group said to be finite and infinite?

(iii) Show that every subgroup of index 2 is a normal subgroup. Define kernel of a group.

(7)

- (iv) What do you mean by reducible and irreducible representations of group? Explain with example.

$$4+(2+2)+(2+2)+4=16$$

5. (a) (i) A pair of coins has thrown. Find the probability of getting two heads. What do you mean by 'equally likely' and 'mutually exclusive' events? Explain with example.

(ii) What is normal distribution? Write the probability function and draw the distribution curve.

(iii) Compute $P(A|B)$, if $P(A \cap B) = 0.32$ and $P(B) = 0.5$.

(iv) The scores of English of five students are given. Find the standard deviation of their scores.

$$55, 47, 68, 59, 46$$

$$(2+3)+(2+2)+3+4=16$$

(OR)

(b) (i) Define measurement of central tendency. Explain the different types of measurement of central tendency with proper example.

(8)

(ii) Set a relation between standard deviation and variance.

(iii) When is it suitable to apply the Poisson distribution? A box contains 12 black balls and 12 red balls. What is the probability of drawing two balls of same colour?

(iv) Two dice are tossed once. Find the probability of getting an even number on the first die and total 8.

$$(1+3)+4+(1+3)+4=16$$

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(OR)